

## EJERCICIOS DE ECUACIONES DIFERENCIALES DESARROLLADOS POR SERIES INFINITAS.

1) Determine la solución de  $y'' - 2xy' + y = 0$

Sustituyendo  $y = \sum_{n=0}^{\infty} c_n x^n$  en la ecuación del diferencial nosotros tenemos

$$\begin{aligned} y'' - 2xy' + y &= \sum_{n=2}^{\infty} n(n-1)c_n x^{n-2} - 2 \sum_{n=1}^{\infty} n c_n x^n + \sum_{n=0}^{\infty} c_n x^n \\ &= \sum_{k=0}^{\infty} (k+2)(k+1)c_k + 2x^k - 2 \sum_{k=1}^{\infty} k c_k x^k + \sum_{k=0}^{\infty} c_k x^k \\ &= 2c_2 + c_0 + \sum_{k=1}^{\infty} [(k+2)(k+1)c_k + 2 - (2k-1)c_k] x^k = 0 \end{aligned}$$

Así

$$\begin{aligned} 2c_2 + c_0 &= 0 \\ (k+2)(k+1)c_k + 2 - (2k-1)c_k &= 0 \end{aligned}$$

y

$$\begin{aligned} c_2 &= -\frac{1}{2}c_0 \\ c_k + 2 &= \frac{2k-1}{(k+2)(k+1)}c_k, \quad k = 1, 2, 3, \dots \end{aligned}$$

Escogiendo  $c_0 = 1$  y  $c_1 = 0$  nosotros encontramos

$$\begin{aligned} c_2 &= -\frac{1}{2} \\ c_3 &= c_5 = c_7 = \dots = 0 \end{aligned}$$

$$c_4 = -\frac{1}{8}$$

$$c_6 = -\frac{7}{336}$$

Para  $C_0 = 0$  y  $C_1 = 1$  nosotros encontramos

$$c_2 = c_4 = c_6 = \dots = 0$$

$$c_3 = \frac{1}{6}$$

$$c_5 = \frac{1}{24}$$

$$c_7 = \frac{1}{112}$$

Y así sucesivamente. Así, dos soluciones son.

$$y_1 = 1 - \frac{1}{2}x^2 - \frac{1}{8}x^4 - \frac{7}{336}x^6 - \dots \quad y \quad y_2 = x + \frac{1}{6}x^3 + \frac{1}{24}x^5 + \frac{1}{112}x^7 + \dots$$

## 2) Determine la solución de $y'' - xy' + 2y = 0$

Sustituyendo  $y = \sum_{n=0}^{\infty} c_n x^n$  en la ecuación del diferencial nosotros tenemos:

$$\begin{aligned} y'' - xy' + 2y &= \sum_{n=2}^{\infty} n(n-1)c_n x^{n-2} - \sum_{n=1}^{\infty} n c_n x^n + 2 \sum_{n=0}^{\infty} c_n x^n \\ &= \sum_{k=0}^{\infty} (k+2)(k+1)c_k + 2x^k - \sum_{k=1}^{\infty} k c_k x^k + 2 \sum_{k=0}^{\infty} c_k x^k \\ &= 2c_2 + 2c_0 + \sum_{k=1}^{\infty} [(k+2)(k+1)c_k + 2 - (k-2)c_k] x^k = 0 \end{aligned}$$

Así

$$2c_2 + 2c_0 = 0$$

$$(k+2)(k+1)c_k + 2 - (k-2)c_k = 0$$

Y

$$c_2 = -c_0$$

Escogiendo  $C_0 = 1$  y  $C_1 = 0$  nosotros encontramos

$$c_2 = -1$$

$$c_3 = c_5 = c_7 = \dots = 0$$

$$c_4 = 0$$

$$c_6 = c_8 = c_{10} = \dots = 0$$

Para  $C_0 = 0$  y  $C_1 = 1$  nosotros encontramos

$$c_2 = c_4 = c_6 = \dots = 0$$

$$c_3 = -\frac{1}{6}$$

$$c_5 = -\frac{1}{120}$$

Y así sucesivamente. Así, dos soluciones son.

$$y_1 = 1 - x^2$$

Y

$$y_2 = x - \frac{1}{6}x^3 - \frac{1}{120}x^5 - \dots$$

### 3) Determinar la solución de $y'' + x^2y' + xy = 0$

Sustituyendo  $y = \sum_{n=0}^{\infty} c_n x^n$  en la ecuación del diferencial nosotros tenemos:

$$\begin{aligned} y'' + x^2y' + xy &= \sum_{n=2}^{\infty} n(n-1)c_n x^{n-2} + \sum_{n=1}^{\infty} n c_n x^{n-1} + \sum_{n=0}^{\infty} c_n x^{n+1} \\ &= \sum_{k=0}^{\infty} (k+2)(k+1)c_k + 2x^k + \sum_{k=2}^{\infty} (k-1)c_k - 1x^k + \sum_{k=1}^{\infty} c_{k-1}x^k \\ &= 2c_2 + (6c_3 + c_0)x + \sum_{k=2}^{\infty} [(k+2)(k+1)c_{k+2} + kc_{k-1}]x^k = 0 \end{aligned}$$

Así

$$c_2 = 6c_3 + c_0 = 0$$

$$(k+2)(k+1)c_{k-2} + kc_{k-1} = 0$$

Y

$$c_3 = -\frac{1}{6}c_0$$

$$c_k + 2 = \frac{k}{(k+2)(k+1)} c_{k-1}, \quad k = 2, 3, 4, \dots$$

Escogiendo  $c_0 = 1$  y  $C_1 = 0$  nosotros encontramos

$$c_3 = -\frac{1}{6}c_0$$

$$c_4 = c_5 = 0$$

$$c_6 = -\frac{1}{45}$$

Para  $C_0 = 0$  y  $C_1 = 1$  nosotros encontramos

$$c_3 = 0$$

$$c_4 = -\frac{1}{6}$$

$$c_5 = c_6 = 0$$

$$c_7 = \frac{5}{252}$$

Y así sucesivamente. Así, dos soluciones son.

$$y_1 = 1 - \frac{1}{6}x^3 + \frac{1}{45}x^6 - \dots \quad y \quad y_2 = x - \frac{1}{6}x^4 + \frac{5}{232}x^7 - \dots$$

$$4) \quad y' + y = 0$$

$$\sum_{n=1}^{\infty} n C_n x^{n-1} + \sum_{n=0}^{\infty} C_n x^n = 0$$

$$k = n - 1 \quad k = n$$

$$\sum_{k=0}^{\infty} (k+1) C_{k+1} x^k + \sum_{k=0}^{\infty} C_k x^k = 0$$

$$\sum_{k=0}^{\infty} x^k [(k+1) C_{k+1} + C_k] = 0 \Rightarrow C_{k+1} = -\frac{C_k}{k+1}$$

$$k = 0 \Rightarrow C_1 = -C_0$$

$$k = 1 \Rightarrow C_2 = \frac{-C_1}{2} = \frac{-(-C_0)}{2} = \frac{C_0}{2}$$

$$k = 2 \Rightarrow C_3 = \frac{-C_2}{3} = \frac{-\left(\frac{C_0}{2}\right)}{3} = \frac{-C_0}{6}$$

$$k = 3 \Rightarrow C_4 = \frac{-C_3}{4} = \frac{-\left(\frac{-C_0}{6}\right)}{4} = \frac{C_0}{24}$$

$$k = 4 \Rightarrow C_5 = \frac{-C_4}{5} = \frac{-\left(\frac{C_0}{24}\right)}{5} = \frac{-C_0}{120}$$

$$y = C_0 - C_0x + \frac{C_0}{2}x^2 - \frac{C_0}{6}x^3 + \frac{C_0}{24}x^4 - \frac{C_0}{120}x^5$$

$$y = C_0 \left[ 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!} \right]$$

$$y = C_0 \sum_{n=0}^{\infty} \frac{(-x)^n}{n!} = C_0 e^{-x}$$

5)  $y' - 2y = 0$

$$\sum_{n=1}^{\infty} nC_n x^{n-1} - 2 \sum_{n=0}^{\infty} C_n x^n = 0$$

$$k = n-1 \quad k = n$$

$$\sum_{k=0}^{\infty} (k+1)C_{k+1}x^k - 2 \sum_{k=0}^{\infty} C_k x^k = 0$$

$$\sum_{k=0}^{\infty} x^k [(k+1)C_{k+1} - 2C_k] = 0 \Rightarrow C_{k+1} = \frac{2C_k}{k+1}$$

$$k = 0 \Rightarrow C_1 = 2C_0$$

$$k = 1 \Rightarrow C_2 = \frac{2C_1}{2} = 2C_0$$

$$k = 2 \Rightarrow C_3 = \frac{2C_2}{3} = \frac{2(2C_0)}{3} = \frac{4C_0}{3}$$

$$k = 3 \Rightarrow C_4 = \frac{2C_3}{4} = \frac{2}{4} \left( \frac{4C_0}{3} \right) = \frac{8C_0}{12}$$

$$k = 4 \Rightarrow C_5 = \frac{2C_4}{5} = \frac{2}{5} \left( \frac{8C_0}{12} \right) = \frac{16C_0}{60}$$

$$y = C_0 + 2C_0x + 2C_0x^2 + \frac{4C_0}{3}x^3 + \frac{8C_0}{12}x^4 + \frac{16C_0}{60}x^5$$

$$y = C_0 \left[ 1 + 2x + 2x^2 + \frac{4}{3}x^3 + \frac{8}{12}x^4 + \frac{16}{60}x^5 \right] = C_0 \sum_{n=0}^{\infty} \frac{(2x)^n}{n!} \Rightarrow y = C_0 e^{2x}$$

6)  $y' - x^2y = 0$

$$\sum_{n=0}^{\infty} nC_n x^{n-1} - x^2 \sum_{n=0}^{\infty} C_n x^n = 0$$

$$\sum_{n=0}^{\infty} nC_n x^{n-1} - \sum_{n=0}^{\infty} C_n x^{n+2} = 0$$

$$C_1 + C_2x + \sum_{n=3}^{\infty} nC_n x^{n-1} - \sum_{n=0}^{\infty} C_n x^{n+2} = 0$$

$$k = n-3 \quad k = n$$

$$\sum_{k=0}^{\infty} (k+3)C_{k+3} x^{k+2} - \sum_{k=0}^{\infty} C_k x^{k+2} = 0$$

$$\sum_{k=0}^{\infty} x^{k+2} [(k+3)C_{k+3} - C_k] = 0 \Rightarrow C_{k+3} = \frac{C_k}{k+3}$$

$$k = 0 \Rightarrow C_3 = \frac{C_0}{3}$$

$$k = 1 \Rightarrow C_4 = \frac{C_1}{4} = 0$$

$$k = 2 \Rightarrow C_5 = \frac{C_2}{5} = 0$$

$$k = 3 \Rightarrow C_6 = \frac{C_3}{6} = \frac{C_0}{18}$$

$$k = 4 \Rightarrow C_7 = \frac{C_4}{7} = 0$$

$$k = 5 \Rightarrow C_8 = \frac{C_5}{8} = 0$$

$$k = 6 \Rightarrow C_9 = \frac{C_6}{9} = \frac{C_0}{162}$$

$$y = C_0 + \frac{C_0}{3}x^3 + \frac{C_0}{18}x^6 + \frac{C_0}{162}x^9$$

$$y = C_0 \left[ 1 + \frac{x^3}{3} + \frac{x^6}{18} + \frac{x^9}{162} \right]$$

$$y = C_0 \sum_{n=0}^{\infty} \frac{1}{n!} \left( \frac{x^3}{3} \right)^n = C_0 e^{x^3/3}$$

7)  $y'' + y = 0$

$$\sum_{n=2}^{\infty} n(n-1)C_n x^{n-2} + \sum_{n=0}^{\infty} C_n x^n = 0$$

$$k = n-2 \quad k = n$$

$$\sum_{k=0}^{\infty} (k+2)(k+1)C_{k+2} x^k - \sum_{k=0}^{\infty} C_k x^k = 0$$

$$\sum_{k=0}^{\infty} x^k [(k+2)(k+1)C_{k+2} + C_k] = 0 \Rightarrow C_{k+2} = \frac{-C_k}{(k+2)(k+1)}$$

$$k = 1 \Rightarrow C_3 = \frac{-C_1}{6}$$

$$k = 2 \Rightarrow C_4 = \frac{-C_2}{12} = \frac{-(-C_0/2)}{12} = \frac{C_0}{24}$$

$$k = 3 \Rightarrow C_5 = \frac{-C_3}{20} = \frac{-(-C_1/6)}{20} = \frac{C_1}{120}$$

$$k = 4 \Rightarrow C_6 = \frac{-C_4}{30} = \frac{-(C_0/24)}{30} = \frac{-C_0}{720}$$

$$k = 5 \Rightarrow C_7 = \frac{-C_5}{42} = \frac{-(C_1/120)}{42} = \frac{-C_1}{5040}$$

$$k = 6 \Rightarrow C_8 = \frac{-C_6}{56} = \frac{-(-C_0/720)}{56} = \frac{C_0}{40320}$$

$$y = C_0 + C_1 x - \frac{C_0}{2} x^2 - \frac{C_1}{6} x^3 + \frac{C_0}{24} x^4 + \frac{C_1}{120} x^5 - \frac{C_0}{720} x^6 - \frac{C_1}{5040} x^7 + \frac{C_0}{40320} x^8$$

$$y_1 = C_0 - \frac{C_0}{2} x^2 + \frac{C_0}{24} x^4 - \frac{C_0}{720} x^6 + \frac{C_0}{40320} x^8$$

$$y_1 = C_0 \left[ 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} \right] = C_0 \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} = C_0 \cos x$$

$$y_2 = C_1 x - \frac{C_1}{6} x^3 + \frac{C_1}{120} x^5 - \frac{C_1}{5040} x^7$$

$$y_2 = C_1 \left[ x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} \right] = C_1 \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} = C_1 \sin x$$

$$y = y_1 + y_2 = C_0 \cos x + C_1 \sin x$$

8) Resuelva la siguiente ecuación diferencial por medio de series:

$$\dot{y} + y = 0$$

- Serie de potencia:

$$y = \sum_{n=0}^{\infty} C_n X^n$$

$$\frac{dy}{dx} = \sum_{n=0}^{\infty} n C_n X^{n-1} = \sum_{n=1}^{\infty} n C_n X^{n-1}$$

$$\dot{y} + y = \sum_{n=1}^{\infty} n C_n X^{n-1} + \sum_{n=0}^{\infty} C_n X^n$$

De tal manera que para la primera serie, tenemos:  $k = n - 1 \Rightarrow n = k + 1$

Y para la segunda serie:  $k = n$

$$\dot{y} + y = \sum_{k=0}^{\infty} (k + 1) C_{k+1} X^k + \sum_{k=0}^{\infty} C_k X^k$$

$$\dot{y} + y = \sum_{k=0}^{\infty} [(k + 1) C_{k+1} + C_k] X^k = 0$$

$$(k + 1) C_{k+1} + C_k = 0$$

$$C_{k+1} = -\frac{C_k}{k+1}$$

$$k=0 \quad C_1 = -C_0$$

$$k=1 \quad C_2 = -\frac{1}{2}C_1 = \frac{1}{2!}C_0$$

$$k=2 \quad C_3 = -\frac{1}{3}C_2 = -\frac{1}{3 \cdot 2!}C_0 = -\frac{1}{3!}C_0$$

$$k=3 \quad C_4 = -\frac{1}{4}C_3 = -\frac{1}{4 \cdot 3!}C_1 = \frac{1}{4!}C_0$$

$$k=4 \quad C_5 = -\frac{C_4}{5} = -\frac{1}{5 \cdot 4!}C_0 = -\frac{1}{5!}C_0$$

$$k=5 \quad C_6 = -\frac{1}{6}C_5 = \frac{1}{6 \cdot 5!}C_0 = \frac{1}{6!}C_0$$

$$k=6 \quad C_7 = -\frac{1}{7}C_6 = -\frac{1}{7 \cdot 6!}C_0 = -\frac{1}{7!}C_0$$

$$y = \sum_{n=0}^{\infty} C_n X^n$$

$$y = C_0 + C_1x + C_2x^2 + C_3x^3 + C_4x^4 + C_5x^5 + C_6x^6 + C_7x^7 \dots$$

$$y = C_0 - C_0 x + \frac{1}{2!} x^2 C_0 - \frac{1}{3!} C_0 x^3 + \frac{1}{4!} C_0 x^4 - \frac{1}{5!} C_0 x^5 + \frac{1}{6!} C_0 x^6 - \frac{1}{7!} C_0 x^7 + \dots$$

$$y = C_0 \left[ 1 - x + \frac{1}{2!} x^2 - \frac{1}{3!} x^3 + \frac{1}{4!} x^4 - \frac{1}{5!} x^5 + \frac{1}{6!} x^6 - \frac{1}{7!} x^7 + \dots \right]$$

$$y = C_0 \sum_{k=0}^{\infty} \frac{(-1)^k \cdot x^k}{k!}$$

9) Resuelva la siguiente ecuación diferencial por medio de series:

$$\dot{y} - 2y = 0$$

$$\frac{dy}{dx} = \sum_{n=0}^{\infty} n C_n x^{n-1} = \sum_{n=1}^{\infty} n C_n x^{n-1}$$

$$\dot{y} - 2y = \sum_{n=1}^{\infty} n C_n x^{n-1} - \sum_{n=0}^{\infty} 2 C_n x^n$$

$$\dot{y} - 2y = C_1 + \sum_{n=2}^{\infty} n C_n x^{n-1} - \sum_{n=0}^{\infty} 2 C_n x^n$$

$$\dot{y} - 2y = C_1 + \sum_{n=2}^{\infty} n C_n x^{n-1} - 2C_0 - \sum_{n=1}^{\infty} 2 C_n x^n$$

$$\dot{y} - 2y = C_1 + \sum_{k=1}^{\infty} (k+1)C_{k+1}x^k - 2C_0 - \sum_{k=1}^{\infty} 2C_k x^k$$

$$\dot{y} - 2y = C_1 - 2C_0 + \sum_{k=1}^{\infty} [(k+1)C_{k+1} - 2C_k]x^k = 0$$

$$C_1 - 2C_0 = 0$$

$$C_1 = 2C_0$$

$$(k+1)C_{k+1} - 2C_k = 0$$

$$C_{k+1} = \frac{2C_k}{k+1}$$

$$k=1 \quad C_2 = \frac{2}{2}C_1 = C_1 = 2C_0$$

$$k=2 \quad C_3 = \frac{2}{3}C_2 = \frac{2}{3}2C_0 = \frac{4}{3!}C_0$$

$$k=3 \quad C_4 = \frac{2}{4}C_3 = \frac{8}{4!}C_0$$

$$k=4 \quad C_5 = \frac{2}{5}C_4 = \frac{16}{5!}C_0$$

$$k=5 \quad C_6 = \frac{2}{6} C_5 = \frac{2 \cdot 16}{6 \cdot 5!} C_0 = \frac{32}{6!} C_0$$

$$y = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + C_4 x^4 + C_5 x^5 + C_6 x^6 + \dots$$

$$y = C_0 + 2C_0 x + 2C_0 x^2 + \frac{4}{3!} C_0 x^3 + \frac{8}{4!} C_0 x^4 + \frac{16}{5!} C_0 x^5 + \frac{32}{6!} C_0 x^6 + \dots$$

$$y = C_0 \left[ 1 + 2x + 2x^2 + \frac{4}{3!} x^3 + \frac{8}{4!} x^4 + \frac{16}{5!} x^5 + \frac{32}{6!} x^6 + \dots \right]$$

10) Resuelva la siguiente ecuación diferencial lineal:

$$\dot{y} - x^2 y = 0$$

$$\frac{dy}{dx} = \sum_{n=0}^{\infty} n C_n x^{n-1} = \sum_{n=1}^{\infty} n C_n x^{n-1}$$

$$\dot{y} - x^2 y = \sum_{n=1}^{\infty} n C_n x^{n-1} - \sum_{n=0}^{\infty} C_n x^{n+2}$$

$$\dot{y} - x^2 y = C_1 + \sum_{n=2}^{\infty} n C_n x^{n-1} - \sum_{n=0}^{\infty} C_n x^{n+2}$$

$$\dot{y} - x^2 y = C_1 + \sum_{n=2}^{\infty} n C_n x^{n-1} - \sum_{n=0}^{\infty} C_n x^{n+2}$$

$$\dot{y} - x^2 y = C_1 + 2x C_2 + \sum_{n=3}^{\infty} n C_n x^{n-1} - \sum_{n=0}^{\infty} C_n x^{n+2}$$

$$\dot{y} - x^2 y = C_1 + 2x C_2 + \sum_{k=2}^{\infty} (k+1) C_{k+1} x^k - \sum_{k=2}^{\infty} C_{k-2} x^k$$

$$\dot{y} - x^2 y = C_1 + 2x C_2 + \sum_{k=2}^{\infty} [(k+1) C_{k+1} - C_{k-2}] x^k$$

$$(k+1) C_{k+1} - C_{k-2} = 0$$

$$C_{k+1} = \frac{C_{k-2}}{k+1}$$

$$C_1 = 0$$

$$2x C_2 = 0 \Rightarrow C_2 = 0$$

$$k=2 \quad C_3 = \frac{C_0}{3} = \frac{1}{3!} C_0$$

$$k=3 \quad C_4 = \frac{1}{4} C_1 = \frac{1}{4} (0) = 0$$

$$k=4 \quad C_5 = \frac{1}{5} C_2 = \frac{1}{5} (0) = 0$$

$$k=5 \quad C_6 = \frac{1}{6} C_3 = \frac{1}{6} \frac{1}{3!} C_0 = \frac{1}{6 \cdot 3!} C_0$$

$$k=6 \quad C_7 = \frac{1}{7} C_4 = \frac{1}{7} (0) = 0$$

$$y = C_0 + C_1 x + C_2 x^2 + C_3 x^3 + C_4 x^4 + C_5 x^5 + C_6 x^6 + C_7 x^7 \dots$$

$$y = C_0 + 0 + 0 + \frac{1}{3!} C_0 x^3 + 0 + 0 + \frac{1}{6 \cdot 3!} C_0 x^6 + 0 + \dots$$

$$y = C_0 \left[ 1 + \frac{1}{3!} x^3 + \frac{1}{6 \cdot 3!} x^6 \right]$$

11. Resolver la ecuación diferencial  $(x-1)y'' + y'$

$$y = \sum_{n=0}^{\infty} c_n x^n$$

Conocemos que:  $\infty$  sustituyendo esta expresión en la ecuación diferencial a resolver tenemos:

$$\begin{aligned} (x-1)y'' + y' &= \underbrace{\sum_{n=2}^{\infty} n(n-1)c_n x^{n-1}}_{k=n-1} - \underbrace{\sum_{n=2}^{\infty} n(n-1)c_n x^{n-2}}_{k=n-2} + \underbrace{\sum_{n=1}^{\infty} n c_n x^{n-1}}_{k=n-1} \\ &= \sum_{k=1}^{\infty} (k+1)k c_{k+1} x^k - \sum_{k=0}^{\infty} (k+2)(k+1)c_{k+2} x^k + \sum_{k=0}^{\infty} (k+1)c_{k+1} x^k \\ &= -2c_2 + c_1 + \sum_{k=1}^{\infty} [(k+1)k c_{k+1} - (k+2)(k+1)c_{k+2} + (k+1)c_{k+1}] x^k = 0. \end{aligned}$$

Entonces:

$$-2c_2 + c_1 = 0$$

$$(k+1)^2 c_{k+1} - (k+2)(k+1)c_{k+2} = 0$$

Y,

$$c_2 = \frac{1}{2}c_1$$

$$c_{k+2} = \frac{k+1}{k+2} c_{k+1}$$

Luego, haciendo  $C_0 = 1$  y  $C_1 = 0$  obtenemos

$$C_2 = C_3 = C_4 = 0$$

Y Haciendo  $C_0 = 0$  y  $C_1 = 1$ , obtenemos:

$$C_2 = \frac{1}{2}, C_3 = \frac{1}{3}, C_4 = \frac{1}{4}$$

Entonces tenemos las siguientes respuestas:

$$y_1 = 1, y_2 = x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4$$